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REFLECTION OF A SHOCK WAVE FROM THE FREE SURFACE OF AN ELASTOPLASTIC BODY

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The present article examines a model of a macroscopically isotropic ideal elastoplastic body with small trains within the framework of the classical dynamic theory of plasticity. The behavior of the body is described by the Prandtl-Reuss equations, the von Mises plasticity condition, and the associated flow law [1].

Our goal is to theoretically study a two-dimensional model problem concerning the propagation and reflection of a shock wave of finite length from a free boundary. The time of impact (contact time) is considered to be finite in regard to the generation of the wave. This time reaches ~10^{-6} sec for a broad range of metals, so that the length of the wave may be several millimeters - in which case it must be taken into account. Here, we will examine weak shock waves, the pressure at the front being of the order of 1 GPa. The waves do not produce phase changes in the substance.

Thus, we will be examining questions relating to a single local reflection of loading and unloading waves - a case corresponding to the actual shock loading of a body. Problems that were similarly formulated and were solved by similar approaches were addressed in [2-5].

Shock-wave profiles in metals have been well studied both theoretically and experimentally [6, 7] and constitute an elastic precursor which is followed by a slower-moving plastic front. The shock wave splits in two, with the formation of a two-front wave configuration. Here, in actual physical processes involving high-speed impact, the amplitude (intensity) of the shock wave initially rapidly increases. It then decreases monotonically to zero - which corresponds to unloading. This circumstance makes it very difficult to construct the unloading wave [8]. As a result, we will henceforth assume that unloading occurs in the form of a certain stepped wave which moves with the speed of the precursor.

Since the stresses and displacement rates change sign in the unloading wave (to tension), we can say that the front of this wave, having "caught up with" the front of the

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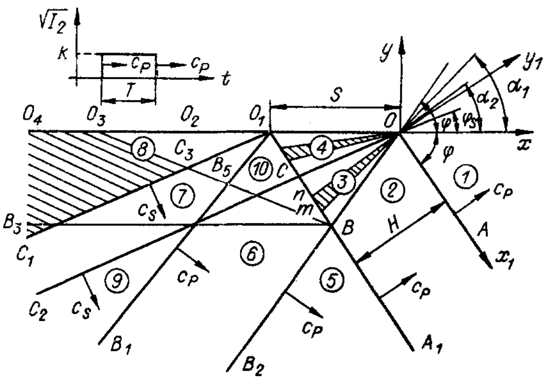

Fig. 1

plastic wave, "reduces" the stresses and displacement rates to their values in the elastic precursor. We will henceforth examine the propagation of an isolated wave of constant pressure. The left corner of Fig. 1 shows an idealized profile of the shock wave that we will be examining.

1. Let (several microseconds after impact) a wave of constant pressure and finite duration T be propagated through a body. The intensity of the shear stresses in the wave

$$I_2 = (1/2)S_{ij}S_{ij} \leqslant k^2 \tag{1.1}$$

(S_{ij} is the stress deviator; k is the yield point of the material). Encountering a free surface at a certain angle φ, the wave is reflected from it back into the body. Figure 1 shows the geometric pattern of the reflection of loading wave OA and unloading wave O_1A_1 forming the shock wave. The problem permits the simplest class of similarity solutions, with a similarity index equal to unity. Proceeding on the basis of well-known results of analysis of the propagation of elastic waves, we postulate the existence of two reflected shock waves at the point O: like-named wave OB and shear wave OC (with the reflection of OA). On the same basis, we also postulate the existence of two reflected shock waves at point O_1: O_1B_1 and O_1C_1 (with the reflection of O_1A_1). The latter two waves have steps of the same form and are needed to satisfy the boundary conditions on the surface. The incident wave OA, of specified intensity Ω = const, propagates in the undeformed medium (zone 1) with the velocity $c^2_p = (\lambda + 2\mu)/\rho$. For plane-strain conditions in the loading region (zones 3 and 4), Baskakov [9] obtained a similarity solution to the dynamical equations of an ideal elastoplastic body in a coordinate system x_1Oy_1 moving at the velocity $c = c_P(\sin\varphi)^{-1}$. The solution was obtained with boundary conditions in stresses on the free surface and with allowance for discontinuities of the stresses and displacement rates in the waves OA, OB, and OC [9]. Here, it was assumed that a constant stress state satisfying condition (1.1) existed in zone 2.

Also presented in [9] was a careful numerical-analytical analysis of possible cases of elastic or plastic deformation of zones 3 and 4. The author determined the intensities of all of the waves, the dimensions of the plastic fans (hatched in Fig. 1) and the velocities of the waves bounding these fans as a function of φ and ν (ν is Poisson's ratio). Thus, the stresses, plastic strains, and displacement rates become known at each point of the wave O_1A_1 on the side of the loading region. The region below O_1A_1 is broken up by reflected waves into characteristic zones 5-10. It will henceforth be convenient to conduct our investigation in moving axes x and y connected with the point O.

Since the reflected waves OB and OC are always located between two neutral or elastic loading regions, then we can assume that their velocities and intensities are constant. This cannot be said of the waves bounding the plastic fans or of the wave O_1A_1, by virtue of the nonuniformity of the plastic-strain distribution in the body ahead of the wave. We will assume that the intensity Ω^* of wave O_1A_1 is different at each point of its front and that it depends on the plastic strains $e_{ij}{}^P$ ahead of the wave. Such a dependence was presented in [10] in standard notation for a plane unloading wave:

$$\delta\Omega^*/\delta t = (\mu/\rho c_P)\, \dot{e}^P_{ij}\nu_i\nu_j. \tag{1.2}$$

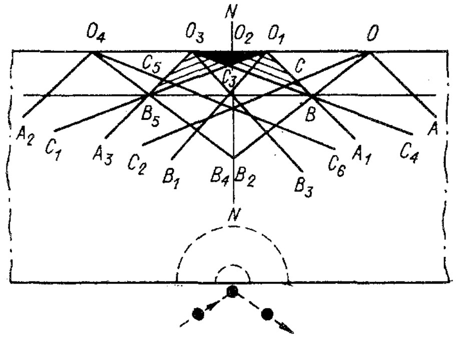

Fig. 2

Replacing δ-differentiation and partial differentiation with respect to time by differentiation with respect to y, we obtain

$$\Omega^* = -\left(c^2/p^2x\sqrt{c^2 - c_P^2}\right)\int_0^y \left(de_{ij}^P/dy\right) y\nu_i\nu_j dy + \Omega_0, \tag{1.3}$$

where integration is performed along the straight line O_1A_1 ($k_1y = -(x + S)$); Ω_0 is the constant of integration, determined from the condition that at point B (in zone 5) $\Omega_B^* = -\Omega$; $B = \left(-\frac{1}{2}S;\ -\frac{1}{2k_1}S\right)$:

$$\Omega_0 = -\left(2c^2/p^2S\sqrt{c^2 - c_P^2}\right)\int_0^{-S/2k_1} \left(de_{ij}^P/dy\right) y\nu_i\nu_j\, dy - \Omega. \tag{1.4}$$

Here, $S = |OO_1| = cT$; $k_1 = \sqrt{c^2 - c_P^2}/c_P = \cot\varphi$; $p^2 = 2(1 - \nu)/(1 - 2\nu)$. It follows in particular from (1.3) that at point O_1

$$\Omega_{O_1}^* = \Omega_0. \tag{1.5}$$

The coordinates x_0, y_0 of the point of intersection of the wave O_1A_1 and any ray $\alpha = \alpha_0(\alpha \in [0;\ \varphi])$, passing through the origin are calculated from the formulas

$$\begin{aligned} x_0 &= -S\sin\varphi\cos\alpha_0/\sin(\alpha_0 + \varphi),\\ y_0 &= -S\sin\varphi\sin\alpha_0/\sin(\alpha_0 + \varphi). \end{aligned} \tag{1.6}$$

Inserting (1.6) into (1.3), we find the intensity of the unloading wave at the given point. The intensities of the waves O_1B_1 and O_1C_1 are constant and are determined from the boundary conditions on the free surface at point O_1. The intensities of the waves BB_2 and CC_2 are also constant for each value of (φ, ν), are known, and are equal respectively to the intensities OB and OC obtained from the elastoplastic solution in the loading zone. The same applies to the wave O_1A_1 at the points of intersection with OB and OC. These conclusions are easily substantiated by using Eqs. (1.7)-(1.8) at points B and C (also see [11]).

The following conditions are satisfied for waves O_1B, O_1B_1, and BB_2

$$[v_i] = \omega\nu_i,\ -c_p[\sigma_{ij}] = \omega(\lambda\delta_{ij} + 2\mu\nu_i\nu_j). \tag{1.7}$$

For waves O_1C_1 and CC_2, we have the relations

$$[v_i]\nu_i = 0,\ [\sigma_{ij}] = -\sqrt{\mu\rho}([v_i]\nu_j + [v_j]\nu_i) \tag{1.8}$$

(ω is the intensity of the corresponding wave; ν_i are components of a unit normal to the wave; the brackets denote discontinuities of the stresses and displacement rates). In particular, $\omega_{O_1B} = \Omega^*(x, y)$. Knowing the stresses and the rates ahead of the waves O_1B and BB_2 (still in zone 5), we can use (1.7)-(1.8) to determine them directly behind the fronts of these waves. Also, we know σ_{12} and σ_{22} on the free surface O_1O_4.

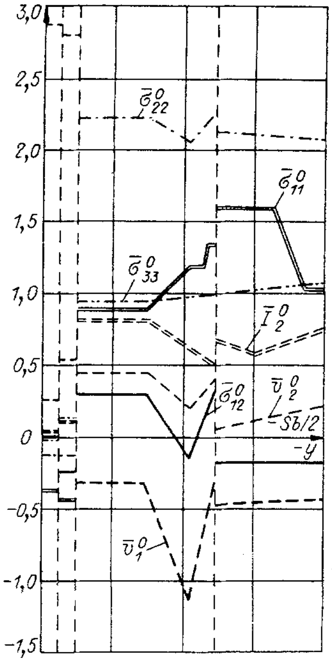

Fig. 3

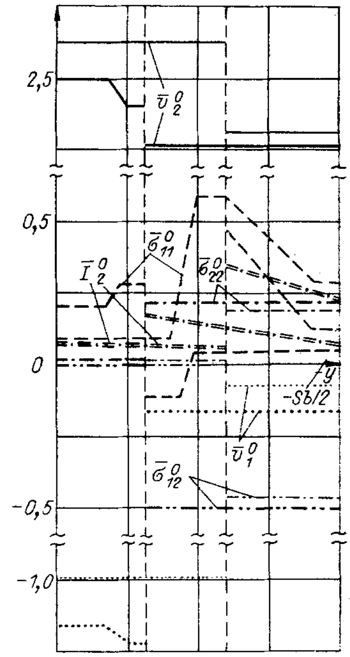

Fig. 4

Thus, we need to construct the complete solution of the problem within the unloading zone $O_4O_1BB_2$ if we know the stresses and the displacement rates at its boundary (see Fig. 1). With this in mind, we applied the method of characteristics to the equations of elasticity theory (henceforth written in the moving variables x and y)

$$\sigma_{ij,j} - \rho\dot{v}_i = 0, \quad \dot{\sigma}_{ij} = \lambda v_{k_1k}\delta_{ij} + \mu(v_{i,j} + v_{j,i}), \tag{1.9}$$

these equations forming a hyperbolic system in the unloading region. It should be noted that the terms $-2\mu\dot{e}_{ij}{}^P$ are absent from the second group of Eqs. (1.9), since we are assuming that the residual plastic strains in the unloading region $e_{ij}{}^P$ = const (otherwise, the calculations are made slightly more complicated). We also drop the equation which determines σ_{33}.

We have five families of characteristics:

$$dy = 0, \ dy/dx = \pm a = \pm \operatorname{tg}\varphi_S, \ dy/dx = \pm b = \pm \operatorname{tg}\varphi. \tag{1.10}$$

Here, $a^2 = \mu(\rho c^2 - \mu)^{-1}$; $b^2 = (\lambda + 2\mu)(\rho c^2 - (\lambda + 2\mu))^{-1}$; $\varphi_S = \arcsin(\sin\varphi/p)$; φ_S is the angle of inclination of the reflected shear shock wave to the free surface. The relations along the characteristics have the form

$$\begin{gathered}
(c/\mu)((p^2 - 2)\sigma_{22} - p^2\sigma_{11}) + 4(1 - p^2)v_1 = f_1(y), \\
(c/\mu)(\sigma_{22} + a(M^2 - 1)\sigma_{12}) + (M^2 - 2)v_1 + (a(M^2 - 1) + \\
+ a^{-1})v_2 = f_2(x + y/a), \\
(c/\mu)(\sigma_{22} - a(M^2 - 1)\sigma_{12}) + (M^2 - 2)v_1 - (a(M^2 - 1) + \\
+ a^{-1})v_2 = f_3(x - y/a), \\
(c/\mu)(\sigma_{22} - b\sigma_{12}) - 2v_1 + b(a^{-2} - 1)v_2 = f_4(x + y/b), \\
(c/\mu)(\sigma_{22} + b\sigma_{12}) - 2v_1 - b(a^{-2} - 1)v_2 = f_5(x - y/b), \\
(p^2 - 2)(\sigma_{11} + \sigma_{22}) - 2(p^2 - 1)\sigma_{33} = f_6(y).
\end{gathered} \tag{1.11}$$

Here, $M^2 = c^2/c_S{}^2$ is the Mach number; $c_S{}^2 = \mu/\rho$; $p^2 = c_P{}^2/c_S{}^2 \geq 2(p^2 = M^2\sin^2\varphi)$. The last relation of (1.11) was obtained directly from the third, fifth, and six equations of system (1.9). Thus, all of the functions f_i are known on the boundary O_1B and BB_2 on the side of the unloading region.

Solving system (1.11) for σ_{ij} and v_i, we find the stresses and the displacement rates at each point within regions 6 and 10 from the formulas

$$
\begin{gathered}
2p^2M^2(c/\mu)\sigma_{11} = (p^2M^2 + 2(p^2 - M^2))(f_4 + f_5) - \\
- 2p^2(f_2 + f_3) - 2M^2 f_1, \\
2M^2(c/\mu)\sigma_{12} = (1 - a^2)a^{-1}(f_2 - f_3) - ((M^2 - 1)a^2 + 1)b^{-1}(f_4 - f_5), \\
2M^2(c/\mu)\sigma_{22} = 2(f_2 + f_3) + (M^2 - 2)(f_4 + f_5); \\
2(p^2 - 1)\sigma_{33} = (p^2 - 2)(\sigma_{11} + \sigma_{22}) + f_6, \\
2M^2 v_1 = (f_2 + f_3) - (f_4 + f_5), \\
2M^2 v_2 = a^2((f_2 - f_3)a^{-1} + (M^2 - 1)(f_4 - f_5)b^{-1}).
\end{gathered} \tag{1.12}
$$

However, these quantities are unknown in zones 7-9, since we do not know the function f_5 in any of them and we do not know the function f_3 in zone 8. To determine them, by using the boundary conditions $\sigma_{12} = \sigma_{22} = 0$ on $0_1 0_4$ we obtain

$$
\begin{gathered}
f_5 = \frac{[2a((M^2 - 1)a^2 + 1) - (M^2 - 2)b(1 - a^2)]f_4 - 4b(1 - a^2)f_2}{b(M^2 - 2)(1 - a^2) + 2a((M^2 - 1)a^2 + 1)}, \\
2f_3 = (2 - M^2)(f_4 + f_5) - 2f_2.
\end{gathered} \tag{1.13}
$$

Inserting (1.13) into (1.12), we find σ_{ij} and v_i in these zones (we recall that f_3 in zones 7 and 9 is determined from (1.11) rather than (1.13)).

Let us examine zone 8. It follows from (1.12)-(1.13) that $\sigma_{12} = \sigma_{22} = 0$ everywhere in this zone. At the same time, v_1 and v_2 are nontrivial both on the free boundary and inside zone 8. We should especially point out the region $0_4 0_3 C_3 C_1$. All f_i except f_1 and f_6 are constant within this region, so that v_1 = const and v_2 = const. This means that the given region moves translationally with a velocity equal to the vector sum of the velocities $\mathbf{v}_1$ and $\mathbf{v}_2$. This may result in microscopic cracking of the material or a local change in the profile of the free surface near the point 0_3. Here, the position of the point 0_3 is determined by the relation $|0_1 0_3| = (b - a)S/2a$. Depending on the stress-strain state of the medium near the wave $0_1 B$ on the loading side, other zones of constant stress and displacement rate in addition to that described above may be formed in the unloading region. For example, it can be concluded on the basis of the results in [9] that the stresses and displacement rates within zones 4, 7, 8, and 10 are constant near point 0_1 but change suddenly with the transition from zone to zone. It should also be noted that direct substitution of stresses (1.12) into (1.1) makes it possible to show whether the medium is deformed elastically or plastically at a given point of the loading zone. The plastic state will probably signify that a transition has occurred from one point of the plasticity surface to another through elastic unloading.

Now let us find the intensity of the waves $0_1 B_1$ and $0_1 C_1$ at point 0_1. We will do this by using Eqs. (1.7) and (1.8). As a result, we can express σ_{ij} and v_i in zones 7, 8, and 10 through the known quantities $\sigma_{ij}^{(4)}$, $v_i^{(4)}$ in zone 4. However, these relations also include the unknowns $\omega_{0_1 B_1}$, $[v_i]_{0_1 C_1}$, which characterize the sought intensities. To find them, we use the boundary condition on the free surface at the point 0_1 $\sigma_{ij}^{(8)} n_j^{(8)} = 0$ ($n_j^{(8)}$ is an inner normal to the free surface) to obtain

$$
\begin{gathered}
p\nu^{(7)}\nu^{(8)}[v_i]_{O_1C_1} = (p^2 - 2)\left(\Omega_0 + \omega_{O_1B_1}\right)\left(n_j^{(8)}\nu_j^{(7)}\nu_i^{(7)} - n_i^{(8)}\right) - \\
- 2\Omega_0\left(\nu_j^{(4)}n_j^{(8)}\nu_i^{(4)} - \nu_k^{(4)}\nu_k^{(7)}\nu_j^{(4)}n_j^{(8)}\nu_i^{(7)}\right) - 2\omega_{O_1B_1}\left(\nu_k^{(10)}n_k^{(8)}\nu_i^{(10)} - \right. \\
\left. - \nu_k^{(10)}\nu_k^{(7)}\nu_j^{(10)}n_j^{(8)}\nu_i^{(7)}\right) + pc_S\mu^{-1}\left(\sigma_{kl}^{(4)}n_l^{(8)}\nu_k^{(7)}\nu_i^{(7)} - \sigma_{ij}^{(4)}n_j^{(8)}\right) \quad (i, j = 1,2);
\end{gathered} \tag{1.14}
$$

$$
\omega_{O_1B_1} = -\xi\Omega_0 - \frac{\left(\sigma_{ij}^{(4)}n_j^{(8)}n_i^{(8)} - \sigma_{ki}^{(4)}n_i^{(8)}\nu_k^{(7)}n_i^{(8)}\nu_i^{(7)}\right)pc_S\mu^{-1}}{(p^2-2)\left[1-2\left(n_k^{(8)}\nu_k^{(7)}\right)^2\right]+2n_k^{(8)}\nu_k^{(10)}\left(n_i^{(8)}\nu_i^{(10)} - 2\nu_i^{(10)}\nu_i^{(7)}\nu_j^{(7)}n_j^{(8)}\right)},
$$

where

$$
\xi = \frac{(p^2 - 2)\left[1 - 2\left(\nu_k^{(7)}n_k^{(8)}\right)^2\right] + 2\nu_k^{(4)}n_k^{(8)}\left(\nu_i^{(4)}n_i^{(8)} - 2\nu_i^{(4)}\nu_i^{(7)}\nu_j^{(7)}n_j^{(8)}\right)}{(p^2 - 2)\left[1 - 2\left(\nu_k^{(7)}n_k^{(8)}\right)^2\right] + 2n_k^{(8)}\nu_k^{(10)}\left(n_i^{(8)}\nu_i^{(10)} - 2\nu_i^{(10)}\nu_i^{(7)}\nu_j^{(7)}n_j^{(8)}\right)}.
$$

This completes the solution of the given problem. Here, the zone affected by unloading wave O_1B can be considered the semi-infinite layer $O_4O_1BB_3$, of thickness $h = (1/2)Sb$ (Fig. 1), rather than the entire region $O_4O_1BB_2$ (which was examined above).

2. In the case of a symmetric pattern of local reflection of a spherical (or cylindrical) shock wave from a free surface (plane) (Fig. 2), zones $O_3C_3B_5$ and O_1C_3B, moving translationally, may result in cleavage of the "triangular cup" $O_3C_3O_1$ as seen in experiments. The occurrence of cleavage failure depends on many factors: the thickness of the plate, the initial pressure pulse, the strain rate, the cleavage stress (tensile strength) of the material. The cleavage stress in turn depends on the conditions of the experiment [12], the angle φ, the nonuniformity of the medium, etc. With the approach being taken here, it can be assumed that cleavage failure at the free surface occurs as a result of the superposition of two solutions on the symmetry axis NN (by virtue of the use of linear equations from the theory of elasticity in the unloading region). Here, we easily determine the dimensions of the cleaved "block:" its radius is equal to $(b - a)S/4a$, while its depth is equal to $(b - a)S/4$. The cleavage time reckoned from the moment when the unloading wave first comes in contact with the free surface is equal to $t_* = (b - a)T/4a$, i.e., it is commensurate with the duration of the shock wave.

3. As an illustration of the material discussed in Sec. 1, we will present a detailed calculation of the stress-strain state of the medium in the unloading region. We will examine the reflection of a limiting shock wave corresponding to the sign of the equality in (1.1). The initial regime entails plastic deformation of the specimen in zone 3 and elastic deformation in zone 4 (see Fig. 1). This situation corresponds to certain values of ν and φ. The angle nOm was small in all of the calculations, not exceeding 3°. Here, $\alpha_1 \approx \varphi$. In accordance with [9], let $\nu = 0.3$, $\varphi = 21°20' = 0.373$, $\alpha_1 = 0.372$, $\alpha_2 = 0.306$. In the coordinate system x_1Oy_1, the dimensionless stresses $\bar\sigma_{ij} = \sigma_{ij}(\sqrt{2}k)^{-1}$ and the displacement rates $\bar v_i = ((\lambda + 2\mu)\rho/2k^2)^{1/2}v_i$ are as follows at the points of intersection of the rays α_1 and α_2 and the unloading wave O_1B in zone 3 and on the wave itself in zone 4:

$$\bar\sigma_{11}^{(3)} = 0{,}04,\ \ \bar\sigma_{12}^{(3)} = -0{,}40,\ \ \bar\sigma_{22}^{(3)} = -1{,}09,\ \ \bar\sigma_{33}^{(3)} = -0{,}32,\quad \bar v_1^{(3)} = -0{,}44,\ \ \bar v_2^{(3)} = 1{,}49; \tag{3.1}$$

$$\bar\sigma_{11}^{(3)} = 0{,}60,\ \ \bar\sigma_{12}^{(3)} = -0{,}36,\ \ \bar\sigma_{22}^{(3)} = -0{,}59,\ \ \bar\sigma_{33}^{(3)} = -0{,}19,\quad \bar v_1^{(3)} = -0{,}42,\ \ \bar v_2^{(3)} = 1{,}68; \tag{3.2}$$

$$\bar\sigma_{11}^{(4)} = -0{,}41,\ \ \bar\sigma_{12}^{(4)} = -0{,}13,\ \ \bar\sigma_{22}^{(4)} = -0{,}12,\ \ \bar\sigma_{33}^{(4)} = -0{,}19,\quad \bar v_1^{(4)} = 0{,}03,\ \ \bar v_2^{(4)} = 1{,}93. \tag{3.3}$$

The initial data is transformed as follows in coordinate system xOy rotated through the angle φ (this system having been used for all of the calculations in Sec. 1)

$$\bar\sigma_{ij}^0 = n_{i\alpha}n_{j\beta}\bar\sigma_{\alpha\beta},\ \ \bar v_1^0 = n_{1i}\bar v_i,\ \ \bar v_2^0 = n_{2i}\bar v_i,\ \ \bar\sigma_{33}^0 = \bar\sigma_{33}, \tag{3.4}$$

where $n_{11} = \cos\varphi$; $n_{12} = \sin\varphi$; $n_{21} = -\sin\varphi$; $n_{22} = \cos\varphi$. The same applies to the components of the plastic strain tensor. Let $\Omega^* = -\Omega\bar\Omega^*$, $\Omega_0 = -\Omega\bar\Omega_0$. We divide (1.3) by $(-\Omega)$. Accordingly, we obtain the following for the three sections of wave O_1B, specifically: 1) O_1n, where $\bar e_{ij}^P = \text{const}$ ($\bar e_{ij}^P = (2\mu/\sqrt{2}k)e_{ij}^P$), 2) nm, where $\bar e_{ij}^P$ changes; 3) mB, where $\bar e_{ij}^P = 0$

$$\bar\Omega_1^* = \bar\Omega_0,\quad \bar\Omega_2^* = \frac{2c_p}{p^2x\Omega\sin 2\varphi}\,\frac{\sqrt{2}k}{2\mu}\int\limits_{y(\alpha_2)}^{y} dy + \bar\Omega_0,\quad \bar\Omega_3^* = 1; \tag{3.5}$$

$$\bar\Omega_0 = \frac{2c_p}{p^2x(\alpha_1)\Omega\sin 2\varphi}\,\frac{\sqrt{2}k}{2\mu}\int\limits_{y(\alpha_2)}^{y(\alpha_1)} dy + 1. \tag{3.6}$$

Here, to make the calculations easier to perform we adopted the following approximation on section nm: $\bar e_{ij}^P = \ln(y/y(\alpha_1))\nu_i\nu_j$. This approximation is consistent with the results obtained in [9]. Also presented in [9] was an expression for the intensity of the wave OA: $\Omega^2 = (3/4)(k^2/\mu^2)p^2c_S^3$. After integration in (3.5)-(3.6), we have

$$\bar{\Omega}_2^* = \sqrt{\frac{2}{3}}\frac{1}{p^2\cos^2\varphi}\frac{\bar{x}(\alpha_2)}{\bar{x}(\alpha_1)}\frac{\bar{x}(\alpha_1)-\bar{x}}{\bar{x}}+1; \tag{3.7}$$

$$\bar{\Omega}_0 = \sqrt{\frac{2}{3}}\frac{\bar{x}(\alpha_1)-\bar{x}(\alpha_2)}{p^2\bar{x}(\alpha_1)\cos^2\varphi}+1. \tag{3.8}$$

Here, $\bar{x} = xS^{-1}(\bar{y} = yS^{-1})$ is calculated from (1.6). Thus, $\bar{x}(\alpha_1)$ is found from the first formula of (1.6) with $\alpha_0 = \alpha_1$, etc. In (3.7), the values of $\bar{x}$ lie within the interval between $\bar{x}(\alpha_1)$ and $\bar{x}(\alpha_2)$.

Conditions (1.7) are satisfied on wave O_1B. In dimensionless form, these conditions are written as

$$[\bar{\sigma}_{ij}^0] = \frac{1}{2}\sqrt{\frac{3}{2}}\bar{\Omega}_{1,2,3}^*((p^2-2)\delta_{ij}+2\nu_i\nu_j); \tag{3.9}$$

$$[\bar{v}_i^0] = -\frac{1}{2}\sqrt{\frac{3}{2}}p^2\bar{\Omega}_{1,2,3}^*\nu_i,\ \nu_i(\sin\varphi, \cos\varphi). \tag{3.10}$$

Thus, on wave O_1B in zones 6 and 10

$$\begin{gathered}
\bar{\sigma}_{11}^0 = \bar{\sigma}_{11}^{0(3,4)} + \frac{1}{2}\sqrt{\frac{3}{2}}\bar{\Omega}_{1,2,3}^*((p^2-2)+2\sin^2\varphi),\\
\bar{\sigma}_{12}^0 = \bar{\sigma}_{12}^{0(3,4)} + \frac{1}{2}\sqrt{\frac{3}{2}}\bar{\Omega}_{1,2,3}^*\sin 2\varphi,\\
\bar{\sigma}_{22}^0 = \bar{\sigma}_{22}^{0(3,4)} + \frac{1}{2}\sqrt{\frac{3}{2}}\bar{\Omega}_{1,2,3}^*((p^2-2)+2\cos^2\varphi),\\
\bar{\sigma}_{33}^0 = \bar{\sigma}_{33}^{0(3,4)} + \frac{1}{2}\sqrt{\frac{3}{2}}\bar{\Omega}_{1,2,3}^*(p^2-2),\\
\bar{v}_1^0 = \bar{v}_1^{0(3,4)} - \frac{1}{2}\sqrt{\frac{3}{2}}p^2\bar{\Omega}_{1,2,3}^*\sin\varphi,\ \bar{v}_2^0 = \bar{v}_2^{0(3,4)} - \frac{1}{2}\sqrt{\frac{3}{2}}p^2\bar{\Omega}_{1,2,3}^*\cos\varphi.
\end{gathered} \tag{3.11}$$

Here, $\bar{\sigma}_{ij}{}^{0(3,4)}$, $\bar{v}_i{}^{0(3,4)}$ are known from (3.1)-(3.4). The discontinuity was determined as $[A] = A^{(6,10)} - \underline{A}^{(3,4)}$. Conditions (1.7) are also satisfied on wave BB_2, but its intensity $\omega_{BB2} = \omega_{OB} = -\bar{\xi}_1\Omega$, where $\bar{\xi}_1 = 3\cos^2 2\varphi - 1 = 0.66$ [9], $\nu_1 = \sin\varphi$, $\nu_2 = -\cos\varphi$. Since $\bar{\sigma}_{ij}{}^{0(5)} = v_i{}^{0(5)} = 0$, then $[\bar{v}_{ij}{}^0] = \bar{\sigma}_{ij}{}^{0(6)}$, $[\bar{v}_i{}^0] = \bar{v}_i{}^{0(6)}$. Thus, in zone 6 on wave BB_2

$$\begin{gathered}
\bar{\sigma}_{11}^0 = \frac{1}{2}\sqrt{\frac{3}{2}}\bar{\xi}_1((p^2-2)+2\sin^2\varphi),\ \bar{\sigma}_{12}^0 = -\frac{1}{2}\sqrt{\frac{3}{2}}\bar{\xi}_1\sin 2\varphi,\\
\bar{\sigma}_{22}^0 = \frac{1}{2}\sqrt{\frac{3}{2}}\bar{\xi}_1((p^2-2)+2\cos^2\varphi),\ \bar{\sigma}_{33}^0 = \frac{1}{2}\sqrt{\frac{3}{2}}\bar{\xi}_1(p^2-2),\\
\bar{v}_1^0 = -\frac{1}{2}\sqrt{\frac{3}{2}}p^2\bar{\xi}_1\sin\varphi,\ \bar{v}_2^0 = \frac{1}{2}\sqrt{\frac{3}{2}}p^2\bar{\xi}_1\cos\varphi.
\end{gathered} \tag{3.12}$$

We find the functions $\bar{f}_i$ $(i = 1, 2, \ldots, 6)$ on wave O_1B on the side of zones 6 and 10 ($\bar{f}_i = (\mu/\sqrt{2}kc)f_i$ $(i = 1, \ldots, 5)$, $\bar{f}_6 = (1/\sqrt{2}k)f_6$), having made use of (1.11) and (3.11):

$$\begin{gathered}
((p^2-2)\bar{\sigma}_{22}^0 - p^2\bar{\sigma}_{11}^0) + (4(1-p^2)/p^2)\sin\varphi\cdot\bar{v}_1^0 = \bar{f}_1,\\
(\bar{\sigma}_{22}^0 + a(\mathrm{M}^2-1)\bar{\sigma}_{12}^0) + ((\mathrm{M}^2-2)\sin\varphi/p^2)\bar{v}_1^0 + ((a(\mathrm{M}^2-\\
-1)+a^{-1})\sin\varphi/p^2)\bar{v}_2^0 = \bar{f}_2,\\
(\bar{\sigma}_{22}^0 - a(\mathrm{M}^2-1)\bar{\sigma}_{12}^0) + ((\mathrm{M}^2-2)\sin\varphi/p^2)\bar{v}_1^0 -\\
-((a(\mathrm{M}^2-1)+a^{-1})\sin\varphi/p^2)\bar{v}_2^0 = \bar{f}_3,\\
(\bar{\sigma}_{22}^0 - b\bar{\sigma}_{12}^0) - (2\sin\varphi/p^2)\bar{v}_1^0 + (b(a^{-2}-1)\sin\varphi/p^2)\bar{v}_2^0 = \bar{f}_4,\\
(\bar{\sigma}_{22}^0 + b\bar{\sigma}_{12}^0) - (2\sin\varphi/p^2)\bar{v}_1^0 - (b(a^{-2}-1)\sin\varphi/p^2)\bar{v}_2^0 = \bar{f}_5,\\
(p^2-2)(\bar{\sigma}_{11}^0 + \bar{\sigma}_{22}^0) - 2(p^2-1)\bar{\sigma}_{33}^0 = \bar{f}_6.
\end{gathered} \tag{3.13}$$

If we insert (3.12) instead of (3.11) into (3.13), we obtain $\bar{f}_i$ on the wave BB_2 on the side of zone 6. Solving system (3.13) for the stresses and displacement rates, we find their values at each point within zones 6 and 10:

$$\begin{aligned}
\bar{\sigma}_{11}^0 &= \left(\frac{1}{2} + \frac{p^2 - M^2}{p^2 M^2}\right)(\bar{f}_4 + \bar{f}_5) - \frac{1}{M^2}(\bar{f}_2 + \bar{f}_3) - \frac{1}{p^2}\bar{f}_1, \\
\bar{\sigma}_{12}^0 &= \frac{1 - a^2}{2aM^2}(\bar{f}_2 - \bar{f}_3) - \frac{1 + (M^2 - 1)a^2}{2bM^2}(\bar{f}_4 - \bar{f}_5), \\
\bar{\sigma}_{22}^0 &= \frac{1}{M^2}(\bar{f}_2 + \bar{f}_3) + \frac{M^2 - 2}{2M^2}(\bar{f}_4 + \bar{f}_5), \\
\bar{\sigma}_{33}^0 &= \frac{p^2 - 2}{2p^2}\left[(\bar{f}_4 + \bar{f}_5) - \frac{1}{(p^2 - 1)}\left(\bar{f}_1 + \frac{p^2}{(p^2 - 2)}\bar{f}_6\right)\right], \\
\bar{v}_1^0 &= \frac{1}{2}\sin\varphi(\bar{f}_2 + \bar{f}_3 - \bar{f}_4 - \bar{f}_5), \\
\bar{v}_2^0 &= \frac{a}{2}\sin\varphi\left[(\bar{f}_2 - \bar{f}_3) + \frac{a(M^2 - 1)}{b}(\bar{f}_4 - \bar{f}_5)\right].
\end{aligned} \tag{3.14}$$

If we take (3.14) and insert $\bar{f}_5$ from (3.13) rather than (1.13), then the desired values become known in zones 7 and 9 as well. If we also determine $\bar{f}_3$ from (1.13), then the sought values of (3.14) are also determined in zone 8. Here, we note that in (1.13) $\bar{f}_i = f_i$ ($i = 2, \ldots, 5$). Numerical calculations were performed on an ES-1060 computer for zones 6, 7, 8, 9, and 10 (in all, we computed 1332 variants of boundary conditions for $\bar{f}_i$). Computing time was 114 sec.

Figures 3 and 4 show some of the results that were obtained: graphs of the change in $\bar{\sigma}_{ij}^0$ and $\bar{v}_i^0$ through the thickness of the layer $O_4O_1BB_3$ (which is approximately one-fifth of S) in the sections x = const, corresponding to points O_1 (Fig. 3), O_2, O_3, and O_4 (Fig. 4). Other sections of the graph are similar. The section x_{O_1} = const was graphed in a small neighborhood of the point O_1 to its left.

All of the curves are satisfactorily approximated by broken lines. The darker lines in Fig. 4 represent the sought values in the section x_{O_2} = const, while the lighter lines represent the same in the section x_{O_3} = const. The vertical dashed curves determine the positions of the shock waves O_1C_1, O_1B_1, CC_2 in the corresponding sections. The stresses and displacement rate undergo discontinuities in these sections, their values being indicated in the figures. Figures 3 and 4 also show the graph of the change in the intensity of the shear stresses $\bar{I}_2^0 = I_2k^{-2}$. It should be noted that inequality (1.1) is written in dimensionless form as $\bar{I}_2^0 = \bar{S}_{ij}\bar{S}_{ij} \leq 1$. After performing the appropriate transformations, we obtain

$$\bar{I}_2^0 = \frac{1}{3}(\bar{\sigma}_{kk}^0)^2 - (\bar{\sigma}_{11}^0\bar{\sigma}_{22}^0 + \bar{\sigma}_{11}^0\bar{\sigma}_{33}^0 + \bar{\sigma}_{22}^0\bar{\sigma}_{33}^0) + \bar{\sigma}_{12}^{0^2} \leqslant \frac{1}{2}.$$

The graphs clearly indicate the loading-zone points at which the material is deformed elastically ($\bar{I}_2^0 < 1/2$), and plastically ($\bar{I}_2^0 > 1/2$). In the sections x_{O_2} = const, x_{O_3} = const, and x_{O_4} = const, the stress $\bar{\sigma}_{33}^0$ is approximately equal to zero and is not shown on the graphs. In the section x_{O_4} = const and to the left, the stresses $\bar{\sigma}_{12}^0$ and $\bar{\sigma}_{22}^0$ are everywhere equal to zero, while $\bar{v}_1^0$ and $\bar{v}_2^0$ are constant and equal to their values in the section x_{O_3} on the free surface. The stress $\bar{\sigma}_{11}^0$ repeats the broken line in the section x_{O_3} = const to the point of discontinuity, but the line is subsequently continuous (this is shown by the fine dashed line in Fig. 4). The value of $\bar{I}_2^0$ changes continuously in this section from $\bar{I}_2^0 = 0.002$ at point O_4 to $\bar{I}_2^0 = 0.02$ at point (x_{O_4}; $-bS/2$). For $\bar{\sigma}_{11}^0$, it follows that both tension ($\bar{\sigma}_{11}^0 > 0$) and compression ($\bar{\sigma}_{11}^0$, < 0) occur at such surface points. Thus, the material is compressed in the neighborhood of point O_1, but with depth it is transformed to the tensile state (as with increasing distance from the point O_1 to the left of the x axis). It is also apparent from an analysis of Figs. 3 and 4 that shear can take place along the line O_3C_3 (see Fig. 1), since the values of $\bar{v}_1^0$ and $\bar{v}_2^0$ are different on both sides of O_3C_3.

Thus, the region of compression of the material up to the yield point (and above) between shock waves OA and O_1A_1 (loading zone) is replaced by a region of "attenuation" of compression or a tensile region after the waves' reflection from the free surface. Large $\bar{\sigma}_{22}^0$ and $\bar{v}_2^0$ under the surface may lead to its normal fracture (microcrack formation) near point O_1 or to cleavage when the technical cohesive strength of the material $\sigma_{22}^0 = \sigma_{cr}$ is exceeded. If we assume that $\sigma_{cr} \sim k \approx 3$ GPa (aluminum), then cleavage can take place in our problem.

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